The Halting Problem

P vs. NP

Halting Problem Motivation...

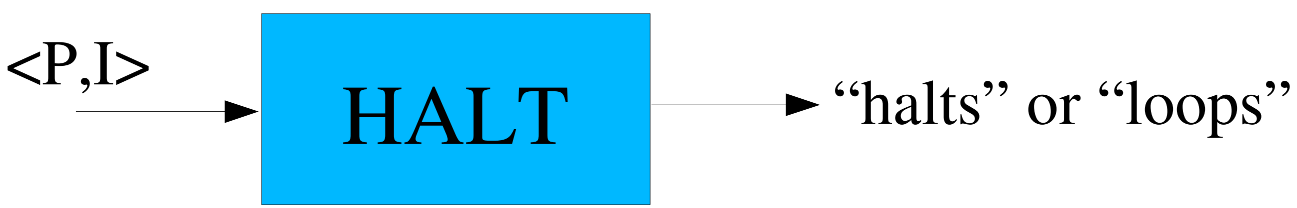
- Infinite loops are a problem:

```
x = 10;
while x < 20,
   x = x – 1;
end
```

- Why don't compilers check for this?
- Can't be done (perfectly). We'll see a proof by contradiction.

HALT program

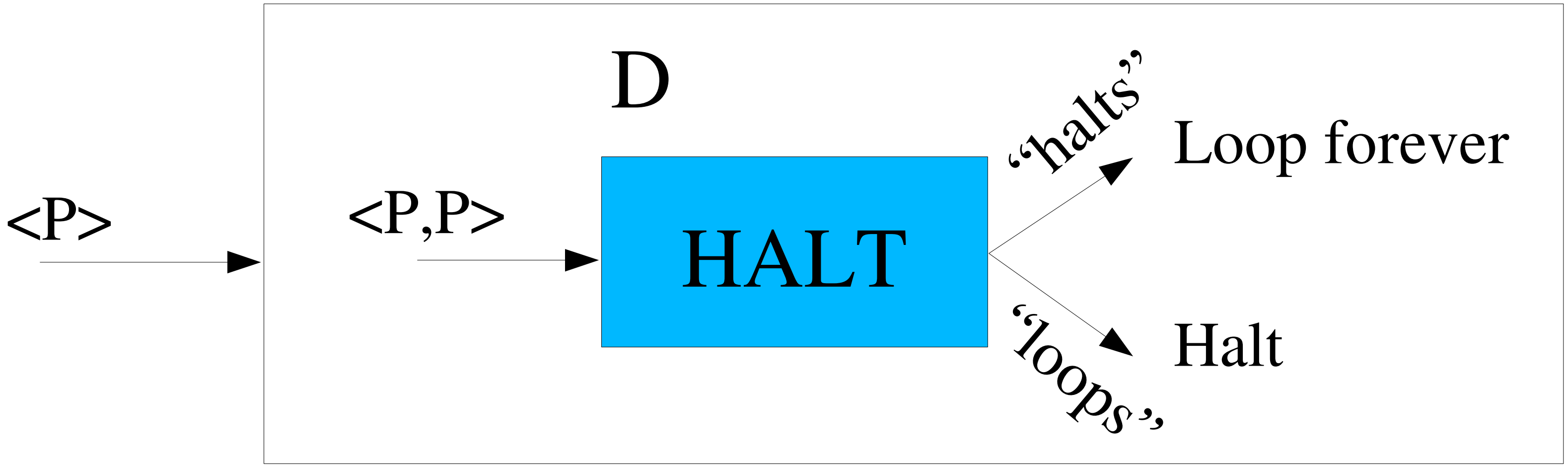
- Assume we have a program HALT:
  - Take a program description, as well as a program input.
  - Returns “halt” if the program halts.
  - Returns “loops” if the program loops forever.



(this follows a proof from Sipser's Introduction to the Theory of Computation, 2006

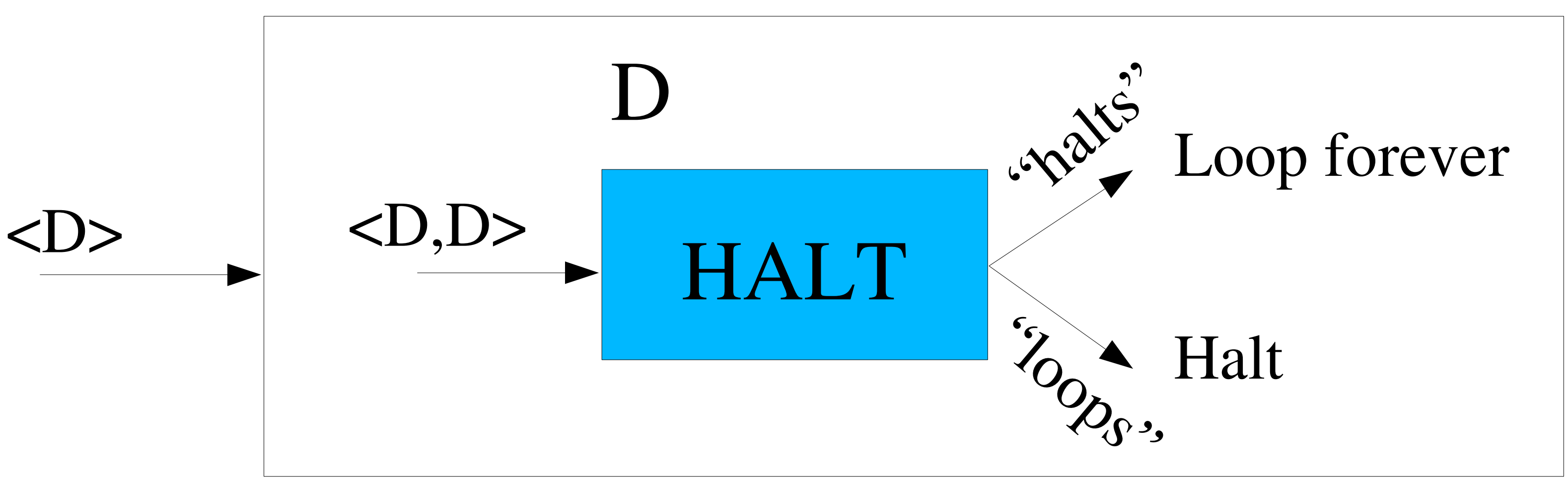
A New Machine...

- Let's create a new program D, that uses HALT as a subroutine.
- D takes a program description as input.
  - Sends the program as both program and input to HALT.
  - If HALT says “halt” D enters an infinite loop.
  - If HALT says “loops” D halts.



The Contradiction...

- Now we pass <D> to the program D.
- What happens?
  - If HALT tells D that D halts on input <D>, then D loops forever on input <D>.
  - If HALT tells D that D loops forever on input <D>, then D halts on input <D>.



What?

- We assumed the existence of a program HALT that can always determine whether a program will halt or run forever.
- We constructed a scenario in which, no matter what answer HALT returns it is wrong.
- Therefore HALT cannot exist.

Rice's Theorem

- Any non-trivial property of programs is undecidable.
- The halting problem is one example among many.

Intractable Problems

- I’ve claimed that the traveling salesperson problem is intractable.
- The best algorithm we’ve seen so far is O(N!).
- How can we know that there isn’t a better algorithm? Maybe O(N)?
- The short answer is that we can’t.
- There is a longer answer that leads us to conclude that TSP is very likely intractable…

P and NP

- P the set of problems that can be solved in polynomial time by some algorithm.
- NP the set of problems that can be solved in polynomial time by a non-deterministic algorithm.
- Non-deterministic??

Non-Determinism

- Deterministic algorithm must *find* a solution.
- Non-deterministic algorithms get to guess a solution, and only need to *check it*.
- Examples:
  - Sorting has a deterministic polynomial time algorithm.
  - TSP-decision problem has a non-deterministic polynomial time algorithm.

What's the Difference?

- Everyone who is anyone believes that P and NP are different sets.
- In other words, there are some problems that can be solved in polynomial time using non-determinism, but can’t be solved in polynomial time deterministically.

An Aside: Reductions

- Problem A is said to be reducible to problem B if
  - An efficient solution to B would yield an efficient solution to A.
- Example:
  - IS-THERE-A-PATH is reducible to FIND-SHORTEST-PATH.
- Informally, if A reduces to B, B is at least as hard as A.

NP-Completeness

- It turns out there is (at least) one problem that all problems in NP reduce to: SAT.
  - SAT is the problem of checking to see if a boolean expression is satisfiable.
- Therefore an efficient solution to SAT would yield an efficient solution to *every* problem in NP.
  - SAT is NP-Complete.
  - SAT is (probably) intractable.
- (We won't prove this...)

Are There Other NP-Complete Problems?

- If so, how would we show it?
- Reductions – Any problem that SAT reduces to is also NP-Complete.
- There are *many* such problems.